

## A note on two-loop superloop

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### Abstract

We explore the duality between supersymmetric Wilson loop on null polygonal contours in maximally supersymmetric Yang-Mills theory and next-to-maximal helicity violating (NMHV) scattering amplitudes. Earlier analyses demonstrated that the use of a dimensional regulator for ultraviolet divergences, induced due to presence of the cusps on the loop, yields anomalies that break both conformal symmetry and supersymmetry. At one-loop order, these are present only in Grassmann components localized in the vicinity of a single cusp and result in a universal function for any number of sites of the polygon that can be subtracted away in a systematic manner leaving a well-defined supersymmetric remainder dual to corresponding components of the superamplitude. The question remains though whether components which were free from the aforementioned supersymmetric anomaly at leading order of perturbation theory remain so once computed at higher orders. Presently we verify this fact by calculating a particular component of the null polygonal super Wilson loop at two loops restricting the contour kinematics to a two-dimensional subspace. This allows one to perform all computations in a concise analytical form and trace the pattern of cancellations between individual Feynman graphs in a transparent fashion. As a consequence of our consideration we obtain a dual conformally invariant result for the remainder function in agreement with one-loop NMHV amplitudes.

The duality between scattering amplitudes in  $\mathcal{N} = 4$  super-Yang-Mills theory and a supersymmetric extension of the Wilson loop spanned on a polygonal closed contour with its sites defined by particles' momenta involved in scattering occupied an important niche in devising new techniques for analysis of dynamics of gauge theories at weak and strong coupling regimes and interpolation between the two. A distinguished role played by the maximal supersymmetry in four dimensions is that all particles of the theory can be combined into a single CPT self-conjugated light-cone superfield  $\Phi$  defined by a (finite) series in the Grassmann variable  $\eta^A$  with coefficients determined by the fields of appropriate helicity to compensate for the deficit introduced by the  $\eta$  itself and matching  $SU(4)$  tensor structure [1, 2]. Thus, the  $n$ -particle S-matrix of the theory is concisely represented by the amputated Green  $\mathcal{A}_n$  functions of  $n$  superfields  $\Phi$ . Extracting the (super)momentum conservation laws allows one to cast the superamplitude  $\mathcal{A}_n$  into the following form [2]

$$\mathcal{A}_n = i(2\pi)^4 \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i) \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \dots \langle n-1n \rangle} \hat{\mathcal{A}}_n(\lambda_i, \tilde{\lambda}_i, \eta_i). \quad (1)$$

The use of the spinor helicity formalism, adopted here and below, simplifies the representation of the amplitude. Namely, the massless particles' momenta  $p_i^{\dot{\alpha}\alpha} = \tilde{\lambda}^{\dot{\alpha}} \lambda^\alpha$  and their chiral charges  $q^{A,\alpha} = \eta^A \lambda^\alpha$  are written by means of commuting Weyl spinors  $\lambda^\alpha$  and  $\tilde{\lambda}^{\dot{\alpha}}$ , with their inner product defined as  $\langle ij \rangle = \lambda_i^\alpha \lambda_{j\alpha}$  and  $[ij] = \tilde{\lambda}_{i\dot{\alpha}} \tilde{\lambda}_j^{\dot{\alpha}}$ , as well as anticommuting Grassmann variables  $\eta^A$  transforming in the fundamental of  $SU(4)$ . The reduced amplitude  $\hat{\mathcal{A}}_n$  admits an expansion in terms of  $\eta$ 's

$$\hat{\mathcal{A}}_n = \hat{\mathcal{A}}_{n,0} + \hat{\mathcal{A}}_{n,1} + \dots, \quad (2)$$

that terminates at order<sup>1</sup>  $k = n - 4$ , with each term being a homogeneous polynomial of degree  $\eta^{4k}$ . Each term in this expansion describes scattering of particle with total helicity  $-n + 4 + 2k$ , with the leading term being the maximal helicity-violating amplitudes (MHV), then the next-to-maximal helicity-violating (NMHV) amplitude etc. At tree level,  $\hat{\mathcal{A}}_{n,0}^{(0)} = 1$ , while the latter can be written as a sum [3]

$$\hat{\mathcal{A}}_{n,1}^{(0)} = \sum_{1 < q < r < n} R_{n;qr}, \quad (3)$$

of superconformal invariants

$$R_{n;qr} = \frac{\delta^4(\langle n, q-1, q, r-1 \rangle \chi_r + \text{cyclic})}{\langle q-1, q, r-1, r \rangle \langle q, r-1, r, n \rangle \langle r-1, r, n, q-1 \rangle \langle r, n, q-1, q \rangle \langle n, q-1, q, r-1 \rangle}, \quad (4)$$

written in terms of momentum twistors  $Z_j^a = (\lambda_j^\alpha, x_j^{\dot{\alpha}\alpha} \lambda_{j\alpha})$ , with angle-brackets being  $\langle ijkl \rangle = \varepsilon_{abcd} Z_i^a Z_j^b Z_k^c Z_l^d$ . In what follows, the focus of our analysis will be  $\hat{\mathcal{A}}_{n,1}$  at higher orders of perturbation theory.

A profound realization of the past few years was that the superamplitude (1) is expected to admit a dual representation in terms of a Wilson superloop [4, 5] spanned on a closed polygonal supercontour with its vertices localized at  $(x_i, \theta_i^A)$  such that its path segments are proportional to the particles' (super)momenta,  $p_i^{\dot{\alpha}\alpha} = (x_1 - x_{i+1})^{\dot{\alpha}\alpha} \equiv x_{i,i+1}^{\dot{\alpha}\alpha}$  and  $q^{A,\alpha} = (\theta_i - \theta_{i+1})^{A,\alpha} \equiv \theta_{ii+1}^{A,\alpha}$ ,

$$\langle \mathcal{W}_n(x_i, \theta_i) \rangle = \frac{1}{N_c} \langle \text{tr} (\mathcal{W}_{[1n]} \dots \mathcal{W}_{[32]} \mathcal{W}_{[21]}) \rangle, \quad (5)$$

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<sup>1</sup>Nilpotence of the Grassmann variables alone is not sufficient to produce this constraint, the reduction by maximal degree by four is a consequence of superconformal symmetry.

where

$$\mathcal{W}_{[i+1,i]} = P \exp \left( ig \int_0^1 dt \mathcal{B}_i(t) \right), \quad (6)$$

where the path is parametrized by  $x_{[ii+1]}(t) = x_i - tx_{ii+1}$ ,  $\theta_{[ii+1]}(t) = \theta_i - t\theta_{ii+1}$  and the first few terms in the superconnection read<sup>2</sup>

$$\mathcal{B}_i(t) = -\frac{1}{2}\langle i|A(t)|i\rangle - \frac{i}{2}\chi_i^A[\bar{\psi}_A(t)|i] - \frac{i}{2}\chi_i^A\left(\frac{1}{2}\langle\theta_{i,i+1}^A(t)|D|i\rangle + \eta_i^B\right)\bar{\phi}_{AB}(t) + \dots \quad (7)$$

The duality relation between the two objects is of the following form

$$\langle \mathcal{W}_{n;k}(x_i, \theta_i) \rangle = \left( \frac{g^2 N_c}{4\pi^2} \right)^k \hat{\mathcal{A}}_{n;k}(\lambda_i, \tilde{\lambda}_i, \eta_i). \quad (8)$$

It is a generalization of the duality for the lowest  $k = 0$  MHV component [6, 7, 8, 9, 10] elucidated by now through multi-loop calculations [11, 12, 13, 14, 15, 16, 17]. The above equation has the unusual property of mixing orders of perturbation theory on both sides of the equation, for instance, the  $\ell$ -th order  $k = 1$  NMHV amplitude emerges from a  $(\ell + 1)$ -loop computation of the superloop, however, from terms quartic in Grassmann variables, etc.

To lowest order in coupling the amplitudes and their duals on the super Wilson loop side are expected to be invariant under the so-called dual superconformal symmetry which acts on the dual coordinates  $(x_i, \theta_i)$  [3]. At subleading orders in coupling, some of the symmetry generators are broken by the ultraviolet regulator in a predictable fashion. This was clearly demonstrated in great details for MHV amplitudes and its dual bosonic Wilson loop in Ref. [9]. However, the first encounter with the superloop's  $\eta^4$ -component, dual to the tree NMHV amplitudes, demonstrated that the former suffer from another anomalous effect [18]. Namely, the use of the Four-Dimensional Helicity scheme [19], adopted for the bulk of higher loop calculations on the amplitudes side as it preserves the spinor-helicity formalism, induces a conformal and supersymmetric anomaly which breaks the above correspondence [18, 20]. However, this anomalous contribution has a universal form and can be subtracted away in a consistent manner, restoring the supersymmetry and conformal symmetry and thus resuscitating the conjectured duality. It is important to realize at the NMHV level, the degree four Grassmannian structure becomes anomalous provided it contains at most three adjacent indices [20], e.g.,  $\chi_{i-1}^2 \chi_i \chi_{i+1}$ ,  $\chi_{i-1}^2 \chi_i^2$  etc. Therefore, any structure where at least one of the indices is not adjacent to the rest will be conformal and given by the corresponding component of the  $R$ -invariants. The question still remains whether those components that were not anomalous at leading order develop unexpected anomalies once computed at subleading orders. This is the issue that we will address in the present study.

We will perform a two-loop computation of a non-anomalous component at leading order, picking  $\chi_2 \chi_3 \chi_6 \chi_7$  as the object of analysis. In order to be able to track explicitly all intricacies of cancellations between Feynman diagrams without the complications of dealing with higher-degree transcendental functions intrinsic to computations in the full four-dimensional kinematics, we will restrict the contour of the superloop to a two-dimensional subspace [6, 21, 22]. In this situation, the highest transcendentality that one can expect in the result is degree two, which encompasses dilogarithms and squares of logs (as well as their lower powers). Moreover, the

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<sup>2</sup>These are the only components that we will need for the main calculation performed in the paper.

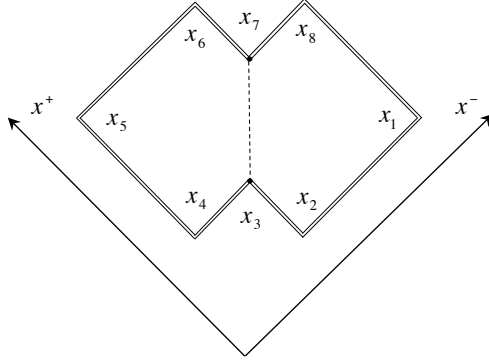


Figure 1: Null octagonal Wilson loop contour in the two-dimensional kinematics. The exchanged scalar between the cusps  $x_3$  and  $x_7$  selects the one-loop  $\chi_2\chi_3\chi_6\chi_7$  component of the superloop and expressible in terms of the corresponding Grassmann projection of the  $R_{8;37}$  superinvariant.

first nontrivial loop has the octagonal shape as shown in Fig. 1, along with the definition of the light-like directions of the segments. The component in question of the tree NMHV amplitude is expressed in terms of  $R_{8;37}$ ,

$$R_{8;37} = \frac{\chi_2\chi_3\chi_6\chi_7}{2\langle 23\rangle\langle 67\rangle x_{73}^+ x_{73}^-} + \dots \quad (9)$$

This results can be easily reproduced by evaluating the corresponding component of the one-loop super Wilson loop, which is determined by the correlation function of two superconnections  $\mathcal{B}$  each of which gets reduced to boundary terms<sup>3</sup> with the scalar field localized at the vertices  $x_3$  and  $x_7$ ,

$$\langle \mathcal{W}_{8,1}^{(1)} \rangle = -\frac{g^2}{4\pi^2} \frac{C_F}{2} \frac{\chi_2\chi_3\chi_6\chi_7}{\langle 23\rangle\langle 67\rangle x_{73}^+ x_{73}^-} (-\mu^2 x_{73}^+ x_{73}^-)^\varepsilon. \quad (10)$$

Here we kept the regularized form of the one-loop result since it will be essential for the definition of the remainder function in the discussion that follows. Notice that we absorbed transcendental constants into the rescaled mass parameter  $2\pi e^{\gamma_E} \mu^2 \rightarrow \mu^2$ . Removing the regulator,  $\varepsilon \rightarrow 0$ , we immediately see that the one-loop superloop is expressible in terms of the  $R_{8;37}$  component of the superconformal invariant. This is the expected result since the anomaly emerges only in adjacent Grassmann components as explained above.

The complexity level of the computation that follows is comparable to the two-loop calculation of the bosonic Wilson loop which is dual to MHV amplitudes. Presently, the two-loop analysis yields the dual to the one-loop NMHV amplitude since we are extracting degree-four Grassmann component. As in our previous studies [18, 20] we will adopt the Four-Dimensional Helicity scheme [19] to regularize divergences in Feynman graphs. This regularization is the closest one to the way one tackles infrared divergent scattering amplitudes. The details of the analysis are deferred to the Appendix.

Due to the choice of the particular Grassmann component, a number of Feynman graphs should not taken into account. Namely, at second order in coupling, one has to include the

<sup>3</sup>For instance,  $\mathcal{W}_{[32]} = \mathcal{W}_{[43]} = 1 + \frac{1}{2}g^2\chi_2^A\chi_3^B\bar{\phi}_{AB}(x_3)/\langle 23\rangle$  and analogously for other two segments adjacent to the vertex  $x_7$ .

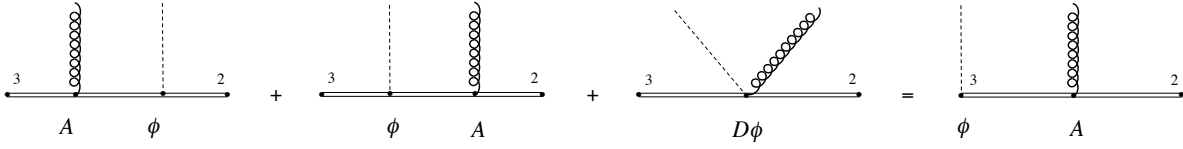


Figure 2: Cancellation mechanism between the different ordering of gluon and scalar emission with the seagull terms stemming from the field component of the covariant derivative  $D\bar{\phi}_{AB}$ .

effects from the covariant derivative (see the last term in Eq. (7)) along with emission of scalar and gluon fields off the super-Wilson links. However, a quick inspection demonstrates that the sum of two orderings of emission along with seagull terms vanish as shown in Fig. 2. The cancellation works as follows. Consider the [32]-superlink as an example. Expanding it to second order in  $g$ , and keeping track of  $\chi_2\chi_3$  component only (and ignoring fermions for a moment), we find

$$\begin{aligned} \mathcal{W}_{[32]} \stackrel{\chi_2\chi_3}{=} & \frac{ig^2}{4} \frac{\chi_2^A \chi_3^B}{\langle 23 \rangle} \left\{ \int_0^1 dt \, t \, \langle 2 | [A(t), \bar{\phi}_{AB}(t)] | 2 \rangle \right. \\ & \left. - \int_0^1 dt \int_0^t dt' \left[ \langle 2 | A(t) | 2 \rangle \left( t' \frac{d}{dt'} + 1 \right) \bar{\phi}_{AB}(t') + \left( t \frac{d}{dt} + 1 \right) \bar{\phi}_{AB}(t) \langle 2 | A(t') | 2 \rangle \right] \right\}. \end{aligned} \quad (11)$$

Here the argument of all functions involved stands for  $f(t) \equiv f(x_{[23]}(t))$ . The first line above displays the gauge field part of the covariant derivative, while the terms involving derivatives in the second line emerge from its flat part. Finally, derivative-free contributions in the integrand of the two-fold integrals come from two ordering of inserting the gluon and the scalar field into the [32]-link. The follow-up simplification of this expression is straightforward and one finds that the scalar fields is nailed down to the vertex at  $x = x_3$  while the gluon is emitted from any point on the link

$$\mathcal{W}_{[32]} \stackrel{\chi_2\chi_3}{=} -\frac{ig^2}{4} \frac{\chi_2^A \chi_3^B}{\langle 23 \rangle} \bar{\phi}_{AB}(x_3) \int_0^1 dt \, \langle 2 | A(t) | 2 \rangle. \quad (12)$$

It takes the form of the leading order scalar emission vertex and a bosonic Wilson segment attached to it. Analogous arguments apply with minor modifications to other superlinks adjacent to the cusps at  $x_3$  and  $x_7$  yielding contributions with the scalar localized at the cusps and gluon strings attached to it.

As a consequence of this consideration, we are left with graphs where the scalar can only spill off the cusps at  $x_3$  and  $x_7$ , thus generating two-loop diagrams shown in Fig. 3 (a-g) along with other attachment of gluons to other segments of the contour to form a gauge-invariant set. Since the same Grassmann structures, either  $\chi_2\chi_3$  or  $\chi_6\chi_7$ , can be induced by fermions emitted off the two adjacent links, to the the given order in coupling, there is an extra graph of the type (h). Notice that it is required by supersymmetry of the superloop and it will be instrumental for the cancellation of the double-pole divergences in the non-abelian color structure  $C_F C_A$ .

To present the result of our analysis, we will strip the dependence on the Grassmann variables and powers of the gauge coupling constant from the component of the super Wilson loop that

we are interested in

$$\langle \mathcal{W}_{8;1}^{(2)} \rangle = \frac{1}{2} \left( \frac{g^2}{4\pi^2} \right)^2 \frac{\chi_2 \chi_3 \chi_6 \chi_7}{\langle 23 \rangle \langle 67 \rangle x_{37}^+ x_{37}^-} \sum_{\alpha} \left( C_F^2 w_{\alpha}^A - \frac{1}{2} C_F C_A w_{\alpha}^{\text{NA}} \right). \quad (13)$$

Here the sum runs over the diagrams displayed in Fig. 3 and split it into abelian and maximally non-abelian color Casimirs. The contribution to the abelian part of the Wilson loop stems from the diagrams in (a), (b), (c), (d) and (e) and the result of rather elementary computations gives

$$w_{(a)}^A = -\frac{1}{2} (-x_{73}^+ x_{73}^- \mu^2)^{\varepsilon} \left[ (-x_{72}^+ x_{83}^- \mu^2)^{\varepsilon} + (-x_{72}^+ x_{87}^- \mu^2)^{\varepsilon} + (-x_{76}^+ x_{74}^- \mu^2)^{\varepsilon} \right. \\ \left. + (-x_{63}^+ x_{74}^- \mu^2)^{\varepsilon} + (-x_{36}^+ x_{34}^- \mu^2)^{\varepsilon} + (-x_{23}^+ x_{83}^- \mu^2)^{\varepsilon} \right] (\varepsilon^{-2} + \zeta_2), \quad (14)$$

$$w_{(c)}^A = -\frac{1}{2} \ln \frac{x_{83}^-}{x_{73}^-} \ln \frac{x_{73}^+}{x_{72}^+} - \frac{1}{2} \ln \frac{x_{74}^-}{x_{73}^-} \ln \frac{x_{73}^+}{x_{63}^+}, \quad (15)$$

$$w_{(b)}^A = w_{(b)}^{\text{NA}}, \quad w_{(d)}^A = w_{(d)}^{\text{NA}}, \quad w_{(e)}^A = w_{(e)}^{\text{NA}}, \quad (16)$$

with  $w_{(b,d,e)}^{\text{NA}}$  displayed below. The abelian part of the expression has the following multiplicative structure

$$\frac{1}{2} \left( \frac{g^2}{4\pi^2} \right)^2 \frac{\chi_2 \chi_3 \chi_6 \chi_7}{\langle 23 \rangle \langle 67 \rangle x_{37}^+ x_{37}^-} \sum_{\alpha} C_F^2 w_{\alpha}^A = \langle \mathcal{W}_{8;1}^{(1)} \rangle \langle \mathcal{W}_{8;0}^{(1)} \rangle, \quad (17)$$

where first factor is the one-loop correction to the superloop  $\langle \mathcal{W}_{8;1}^{(1)} \rangle$  defining the tree NMHV amplitude and the second is the one-loop correction to the bosonic loop which is equal to  $C_F \sum_{\alpha} w_{\alpha}^A$  up to factors of the coupling constant. Thus the remainder function, defined by subtracting the ultraviolet divergent contributions conventionally by

$$\mathcal{R}_{8;1}^{(2)} = \langle \mathcal{W}_{8;1}^{(2)} \rangle - \langle \mathcal{W}_{8;1}^{(1)} \rangle \langle \mathcal{W}_{8;0}^{(1)} \rangle, \quad (18)$$

is solely defined by the maximally nonabelian color. Therefore, the sum of all corresponding contributions has to be dual conformally invariant.

The analysis of the maximally non-abelian contributions is more involved. The result of rather lengthy calculations can be cast to the following form

$$w_{(b)}^{\text{NA}} = -\frac{1}{2} (-x_{73}^+ x_{73}^- \mu^2)^{\varepsilon} \left[ (-x_{23}^+ x_{34}^- \mu^2)^{\varepsilon} + (-x_{76}^+ x_{87}^- \mu^2)^{\varepsilon} \right] (\varepsilon^{-2} + \zeta_2), \quad (19)$$

$$w_{(d)}^{\text{NA}} = \frac{1}{2} \ln \frac{x_{72}^+}{x_{62}^+} \ln \frac{x_{73}^-}{x_{78}^-} + \frac{1}{2} \ln \frac{x_{76}^+}{x_{73}^+} \ln \frac{x_{84}^-}{x_{74}^-} + \frac{1}{2} \ln \frac{x_{63}^+}{x_{62}^+} \ln \frac{x_{73}^-}{x_{34}^-} + \frac{1}{2} \ln \frac{x_{23}^+}{x_{73}^+} \ln \frac{x_{84}^-}{x_{83}^-}, \quad (20)$$

$$w_{(e)}^{\text{NA}} = \frac{1}{2} \ln \frac{x_{62}^+}{x_{67}^+} \ln \frac{x_{78}^-}{x_{48}^-} + \frac{1}{2} \ln \frac{x_{62}^+}{x_{23}^+} \ln \frac{x_{34}^-}{x_{84}^-}, \quad (21)$$

$$w_{(f)}^{\text{NA}} = \varepsilon^{-1} (-x_{73}^+ x_{73}^- \mu^2)^{2\varepsilon} \left[ \frac{1}{2} \left[ 1 - \frac{x_{73}^+}{2x_{23}^+} \right] \ln \frac{x_{72}^+}{x_{73}^+} + \frac{1}{2} \left[ 1 - \frac{x_{73}^+}{2x_{76}^+} \right] \ln \frac{x_{63}^+}{x_{73}^+} \right. \\ \left. + \frac{1}{2} \left[ 1 + \frac{x_{73}^-}{2x_{87}^-} \right] \ln \frac{x_{83}^-}{x_{73}^-} + \frac{1}{2} \left[ 1 + \frac{x_{73}^-}{x_{34}^-} \right] \ln \frac{x_{74}^-}{x_{73}^-} + 2 \right] \quad (22)$$

$$\begin{aligned}
& + \frac{x_{73}^+}{2x_{23}^+} \text{Li}_2 \left( \frac{x_{23}^+}{x_{73}^+} \right) + \frac{x_{73}^+}{2x_{76}^+} \text{Li}_2 \left( \frac{x_{76}^+}{x_{73}^+} \right) - \frac{x_{73}^-}{2x_{34}^-} \text{Li}_2 \left( \frac{x_{43}^-}{x_{73}^-} \right) - \frac{x_{73}^-}{2x_{87}^-} \text{Li}_2 \left( \frac{x_{78}^-}{x_{73}^-} \right) - 2 \\
& + \frac{1}{4} \left[ 1 - \frac{x_{73}^+}{2x_{23}^+} \right] \ln^2 \frac{x_{72}^+}{x_{73}^+} + \frac{1}{4} \left[ 1 - \frac{x_{73}^+}{2x_{76}^+} \right] \ln^2 \frac{x_{63}^+}{x_{73}^+} \\
& + \frac{1}{4} \left[ 1 + \frac{x_{73}^-}{2x_{87}^-} \right] \ln^2 \frac{x_{83}^-}{x_{73}^-} + \frac{1}{4} \left[ 1 + \frac{x_{73}^-}{2x_{34}^-} \right] \ln^2 \frac{x_{74}^-}{x_{73}^-}, \\
w_{(g)}^{\text{NA}} = & \varepsilon^{-1} (-x_{73}^+ x_{73}^- \mu^2)^\varepsilon \left[ \frac{1}{4} (-x_{72}^+ x_{73}^- \mu^2)^\varepsilon \frac{x_{73}^+}{x_{23}^+} \ln \frac{x_{72}^+}{x_{73}^+} + \frac{1}{4} (-x_{63}^+ x_{73}^- \mu^2)^\varepsilon \frac{x_{73}^+}{x_{76}^+} \ln \frac{x_{63}^+}{x_{73}^+} \right. \\
& \left. - \frac{1}{4} (-x_{73}^+ x_{83}^- \mu^2)^\varepsilon \frac{x_{73}^-}{x_{87}^-} \ln \frac{x_{83}^-}{x_{73}^-} - \frac{1}{4} (-x_{73}^+ x_{74}^- \mu^2)^\varepsilon \frac{x_{73}^-}{x_{34}^-} \ln \frac{x_{74}^-}{x_{73}^-} \right] \\
& - \frac{1}{8} \frac{x_{73}^+}{x_{23}^+} \ln^2 \frac{x_{72}^+}{x_{73}^+} - \frac{1}{8} \frac{x_{73}^-}{x_{78}^-} \ln^2 \frac{x_{83}^-}{x_{73}^-} + \frac{1}{8} \frac{x_{73}^+}{x_{67}^+} \ln^2 \frac{x_{63}^+}{x_{73}^+} + \frac{1}{8} \frac{x_{73}^-}{x_{34}^-} \ln^2 \frac{x_{74}^-}{x_{73}^-} - \zeta_2 \\
& - \frac{1}{2} \ln \frac{x_{23}^+ x_{83}^-}{x_{73}^+ x_{73}^-} \ln \frac{x_{73}^+ x_{78}^-}{x_{72}^+ x_{73}^-} + \frac{1}{2} \ln \frac{x_{36}^+ x_{34}^-}{x_{73}^+ x_{73}^-} \ln \frac{x_{73}^+ x_{47}^-}{x_{67}^+ x_{73}^-} + \ln \frac{x_{38}^-}{x_{73}^-} \ln \frac{x_{78}^-}{x_{73}^-} + \ln \frac{x_{63}^+}{x_{73}^+} \ln \frac{x_{67}^+}{x_{73}^+} \\
& + \left[ 1 - \frac{x_{73}^+}{2x_{23}^+} \right] \text{Li}_2 \left( \frac{x_{23}^+}{x_{73}^+} \right) + \left[ 1 - \frac{x_{73}^+}{2x_{76}^+} \right] \text{Li}_2 \left( \frac{x_{76}^+}{x_{73}^+} \right) \\
& + \left[ 1 + \frac{x_{73}^-}{2x_{87}^-} \right] \text{Li}_2 \left( \frac{x_{78}^-}{x_{73}^-} \right) + \left[ 1 + \frac{x_{73}^-}{2x_{34}^-} \right] \text{Li}_2 \left( \frac{x_{43}^-}{x_{73}^-} \right),
\end{aligned} \tag{23}$$

$$\begin{aligned}
w_{(h)}^{\text{NA}} = & \frac{1}{2} (-x_{73}^+ x_{73}^- \mu^2)^\varepsilon (-x_{23}^+ x_{34}^- \mu^2)^\varepsilon \left[ \varepsilon^{-2} + \zeta_2 - \varepsilon^{-1} \ln \frac{x_{72}^+ x_{74}^-}{x_{73}^+ x_{73}^-} \right] \\
& + \frac{1}{2} (-x_{73}^+ x_{73}^- \mu^2)^\varepsilon (-x_{76}^+ x_{87}^- \mu^2)^\varepsilon \left[ \varepsilon^{-2} + \zeta_2 - \varepsilon^{-1} \ln \frac{x_{63}^+ x_{83}^-}{x_{73}^+ x_{73}^-} \right] \\
& - \frac{1}{4} \ln^2 \frac{x_{72}^+ x_{73}^-}{x_{73}^+ x_{74}^-} - \frac{1}{4} \ln^2 \frac{x_{63}^+ x_{73}^-}{x_{73}^+ x_{83}^-} - \text{Li}_2 \left( \frac{x_{23}^+}{x_{73}^+} \right) - \text{Li}_2 \left( \frac{x_{43}^-}{x_{73}^-} \right) - \text{Li}_2 \left( \frac{x_{87}^-}{x_{73}^-} \right) - \text{Li}_2 \left( \frac{x_{76}^+}{x_{73}^+} \right),
\end{aligned} \tag{24}$$

$$w_{(i)}^{\text{NA}} = -\varepsilon^{-1} (-x_{73}^+ x_{73}^- \mu^2)^{2\varepsilon} - 2. \tag{25}$$

Summing up the diagrams, we find that, as anticipated, that all ultraviolet poles cancel among different graphs in the remainder function. In particular, the fermionic graph (h) exactly cancels the maximally non-abelian color structure in diagram (b). The sum of single poles vanish as well provided we set  $(-x_{ij}^+ x_{kl}^- \mu^2)^\varepsilon \rightarrow 1$ , but otherwise they induce finite contributions. The latter are of paramount importance in removing all terms proportional to squares of the logs that depend solely either on plus or minus components of the dual coordinates. Finally, all dilogarithms present in individual graphs sum-up to zero in the remainder function as well. As a result the latter takes a very simple form

$$\begin{aligned}
\mathcal{R}_{8;1}^{(2)} = & \left( \frac{g^2}{4\pi^2} \right)^2 \frac{C_F C_A}{8} \frac{\chi_2 \chi_3 \chi_6 \chi_7}{\langle 23 \rangle \langle 67 \rangle x_{73}^+ x_{73}^-} \\
& \times \left[ \ln u^+ \ln u^- + \ln u^+ \ln(1 + u^-) + \ln(1 + u^+) \ln u^- - \ln(1 + u^+) \ln(1 + u^-) + 2\zeta_2 \right],
\end{aligned} \tag{26}$$

upon the introduction of the conformal cross-ratios

$$u^+ = \frac{x_{32}^+ x_{67}^+}{x_{62}^+ x_{73}^+}, \quad u^- = \frac{x_{87}^- x_{34}^-}{x_{84}^- x_{73}^-}. \tag{27}$$

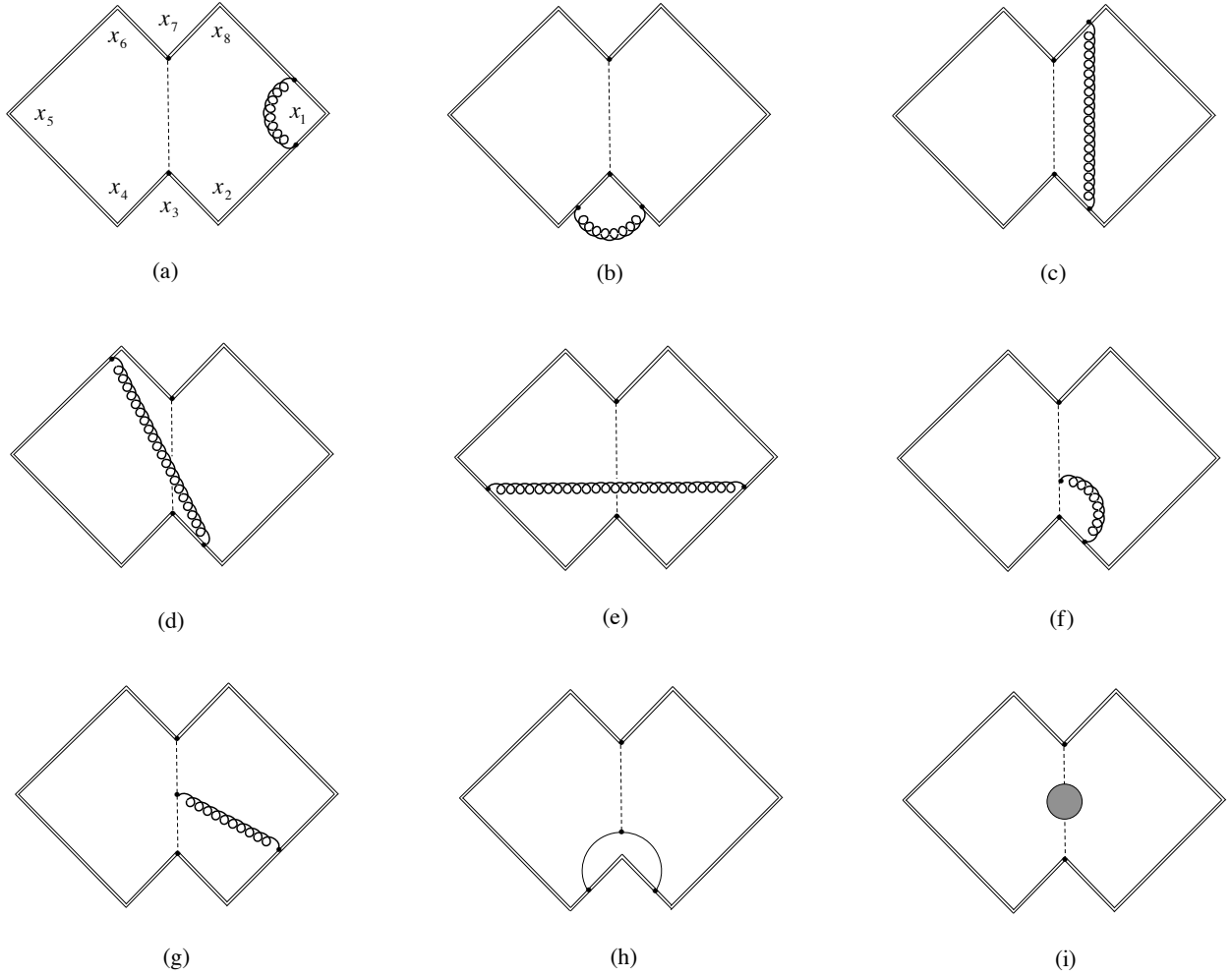


Figure 3: All topologies of Feynman diagrams contributing to the  $\chi_2\chi_3\chi_6\chi_7$  component of the supersymmetric Wilson loop at two loop order. The blob on the scalar line in (i) stands for the sum of vacuum polarization bubbles due to gauge fields and gauginos.

This expression is in agreement with the result of a recent analysis that bypasses the calculation of the Feynman graphs and finds the result in question by integrating the Ward identities associated with  $\bar{Q}$  supersymmetry from the tree-level NNMHV amplitude [23].

Presently, we verified by a brute-force Feynman graph calculation the duality between the supersymmetric extension of the null polygonal Wilson loop and the superamplitude in maximally supersymmetric gauge theory. Our analysis elucidates the validity of the correspondence for Grassmann components which do not involve at least three adjacent particle indices. The latter were shown to be anomalous already at one-loop order. However, once the universal conformal anomaly is subtracted out, the duality gets restored. In the forthcoming work [24], we will demonstrate how one can perform a super-gauge transformation on the super Wilson loop in order to gauge away in a systematic manner the notorious anomalous contributions.

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## A Details of the calculation

The bulk of graphs is easy to compute. Let us pay special attention to a couple of them that are not as straightforward, namely diagrams (g) and (h) in Fig. 3.

### A.1 Diagram (g)

Using the usual Feynman rules, we find the following integral representation for the graph (g),

$$\mathcal{W}_{8;1(h)}^{(2)} = i \frac{g^4 C_F C_A}{(4\pi^{2-\varepsilon})^3} x_{12}^- \frac{\chi_2 \chi_3 \chi_6 \chi_7}{\langle 23 \rangle \langle 67 \rangle} \Gamma^2(1-\varepsilon) \Gamma(2-\varepsilon) \int_0^1 dt J_g(t),$$

where

$$J_g(t) = \int d^{4-2\varepsilon} z \frac{1}{[-(z-x_{[12]}(t))^2]^{1-\varepsilon} [-(z-x_3)^2]^{1-\varepsilon} [-(z-x_7)^2]^{1-\varepsilon}} \left[ \frac{(z-x_7)^+}{(z-x_7)^2} - \frac{(z-x_3)^+}{(z-x_3)^2} \right]. \quad (28)$$

As a first step, we use the Feynman parametrization to put  $J_g$  into the form

$$J_g(t) = - \frac{i\pi^{2-\varepsilon} \Gamma(2-2\varepsilon)}{\Gamma^2(1-\varepsilon) \Gamma(2-\varepsilon)} \times \mu^{4\varepsilon} \int_0^1 \frac{ds_1 ds_2 ds_3 \delta(\sum_{i=1}^3 s_i - 1) (s_1 s_2 s_3)^{-\varepsilon}}{[-s_1 s_2 x_{[12]7}^2 - s_1 s_3 x_{[12]3}^2 - s_2 s_3 x_{73}^2]^{2-2\varepsilon}} [s_1 s_3 x_{[12]3}^+ - s_1 s_2 x_{[12]7}^+ + 2s_2 s_3 x_{73}^+]. \quad (29)$$

Now, expressing  $s_3$  in terms of the other two variables via the  $\delta$ -function constraint and changing the integration variables as<sup>4</sup>  $s_2 \rightarrow \bar{s}_1 s_2$  and as a consequence  $s_3 \rightarrow \bar{s}_1 \bar{s}_2$ , we can integrate with respect to  $t$  to get

$$\begin{aligned} \int_0^1 dt J_g(t) &= \frac{i\pi^{2-\varepsilon} \Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon) \Gamma(2-\varepsilon)} \frac{(-2\mu^2)^{2\varepsilon}}{4x_{12}^-} \int_0^1 \frac{ds_1}{s_1^{1+\varepsilon}} \int_0^1 \frac{ds_2}{(s_2 \bar{s}_2)^\varepsilon} \frac{s_1 \bar{s}_2 x_{23}^+ - s_1 s_2 x_{17}^+ + 2\bar{s}_1 s_2 \bar{s}_2 x_{73}^+}{s_2 x_{17}^+ + \bar{s}_2 x_{23}^+} \\ &\times \left[ \frac{1}{[s_1 s_2 x_{71}^+ x_{71}^- + s_1 \bar{s}_2 x_{12}^- x_{23}^+ + \bar{s}_1 s_2 \bar{s}_2 x_{73}^+ x_{73}^-]^{1-2\varepsilon}} - \frac{1}{s_2^{1-2\varepsilon} (x_{73}^-)^{1-2\varepsilon}} \frac{1}{[s_1 x_{71}^+ + \bar{s}_1 \bar{s}_2 x_{73}^+]^{1-2\varepsilon}} \right]. \end{aligned} \quad (30)$$

It might appear that there is a double pole emerging from this integrals, however, the only pole that realizes is from the  $s_2$  integration in the vicinity of  $s_2 = 0$ , while the expression in curly brackets scales as  $s_1$  and it tends to zero canceling the potential singular behavior. We extract the pole in the  $s_2$  integral via the following formula

$$\int_0^1 \frac{ds_2}{s_2^{1-\varepsilon}} f(s_2) = \varepsilon^{-1} f(0) + \int_0^1 \frac{ds_2}{s_2^{1-\varepsilon}} [f(s_2) - f(0)]. \quad (31)$$

---

<sup>4</sup>Here and below  $\bar{s} \equiv 1 - s$  for any variables.

This allows us to cast the result after some manipulations into the form

$$\int_0^1 dt J_g(t) = -\frac{i\pi^{2-\varepsilon}\Gamma(1-2\varepsilon)}{4\Gamma^2(1-\varepsilon)\Gamma(2-\varepsilon)} \frac{1}{x_{12}^- x_{73}^-} \left[ \varepsilon^{-1} (-2\mu^2 x_{73}^-)^{2\varepsilon} I_1 + (x_{73}^- - 2x_{71}^-) I_2 + x_{73}^+ I_3 \right], \quad (32)$$

with the set of  $I_i$  integrals that can be evaluated with the result

$$\begin{aligned} I_1 &= \int_0^1 \frac{ds_1}{s_1^\varepsilon} \frac{1}{[x_{71}^+ s_1 + x_{73}^+ \bar{s}_1]^{1-2\varepsilon}} \\ &= (-2\mu^2 x_{73}^-)^{-2\varepsilon} \frac{1}{x_{23}^+} \left[ (-2\mu^2 x_{71}^+ x_{73}^-)^\varepsilon (-2\mu^2 x_{73}^+ x_{73}^-)^\varepsilon \ln \frac{x_{71}^+}{x_{73}^+} - \varepsilon \text{Li}_2 \left( \frac{x_{23}^+}{x_{73}^+} \right) \right], \end{aligned} \quad (33)$$

$$\begin{aligned} I_2 &= \int_0^1 ds_1 ds_2 \frac{1}{x_{71}^+ x_{71}^- s_1 s_2 + x_{23}^+ x_{12}^- s_1 \bar{s}_2 + x_{73}^+ x_{73}^- \bar{s}_1 s_2 \bar{s}_2} \\ &= \frac{1}{x_{71}^+ x_{73}^- - x_{73}^+ x_{71}^-} \left[ \ln \frac{x_{73}^+ x_{71}^-}{x_{71}^+ x_{73}^-} \ln \frac{x_{23}^+ x_{12}^-}{x_{73}^+ x_{73}^-} + 2\text{Li}_2 \left( \frac{x_{12}^-}{x_{73}^-} \right) - 2\text{Li}_2 \left( \frac{x_{23}^-}{x_{73}^-} \right) \right], \end{aligned} \quad (34)$$

$$I_3 = \int_0^1 ds_1 ds_2 \frac{\bar{s}_1}{[x_{71}^+ s_1 + x_{73}^+ \bar{s}_1][x_{71}^+ s_1 + x_{73}^+ \bar{s}_1 \bar{s}_2]} = -\frac{1}{x_{23}^+ x_{73}^+} \left[ \ln^2 \frac{x_{71}^+}{x_{73}^+} + 2\text{Li}_2 \left( \frac{x_{23}^+}{x_{73}^+} \right) \right]. \quad (35)$$

Notice that the second integral involves a denominator that mixes both plus and minus components. This effects disappears one we add a mirror symmetric diagram yielding a factorized product of function of plus and minus variables. Summing all diagrams of this topology we get the expression in Eq. (23).

## A.2 Diagram (h)

Now, we turn to the second graph.

$$\mathcal{W}_{8;1(h)}^{(2)} = -i \frac{g^4 C_F C_A}{(4\pi^{2-\varepsilon})^3} \chi_2 \chi_3 \chi_6 \chi_7 \frac{[23]}{\langle 67 \rangle} \Gamma(1-\varepsilon) \Gamma^2(2-\varepsilon) \int_0^1 ds \int_0^1 dt J_h(t, s),$$

where we have used the identity  $[2|(z - x_{23}(t))(z - x_{[34]}(s))|3] = [23](z - x_3)^2$  in order to define the coordinate integral

$$J_h(t, s) = \mu^{4\varepsilon} \int d^{4-2\varepsilon} z \frac{(z - x_3)^2}{[-(z - x_7)^2]^{1-\varepsilon} [-(z - x_{[23]}(t))^2]^{2-\varepsilon} [-(z - x_{[34]}(s))^2]^{2-\varepsilon}}. \quad (36)$$

By means of the standard Feynman parametrization, one can cast it in the form after integration over  $z$

$$J_h(t, s) = -\frac{i\pi^{2-\varepsilon}\Gamma(2-2\varepsilon)}{\Gamma(1-\varepsilon)\Gamma^2(2-\varepsilon)} [4(1-\varepsilon) x_{73}^+ x_{73}^- I_1(t, s) - I_2(t, s)], \quad (37)$$

where

$$I_1(t, s) = \mu^{4\varepsilon} \int_0^1 \frac{ds_1 ds_2 ds_3 \delta(\sum_{i=1}^3 s_i - 1) (s_1 s_2 s_3)^{1-\varepsilon}}{[-s_1 s_2 x_{7[23]}^2 - s_1 s_3 x_{7[34]}^2 - s_2 s_3 x_{[23][34]}^2]^{3-2\varepsilon}}, \quad (38)$$

$$I_2(t, s) = \varepsilon \mu^{4\varepsilon} \int_0^1 \frac{ds_1 ds_2 ds_3 \delta(\sum_{i=1}^3 s_i - 1) s_1^{-\varepsilon} (s_2 s_3)^{1-\varepsilon}}{[-s_1 s_2 x_{7[23]}^2 - s_1 s_3 x_{7[34]}^2 - s_2 s_3 x_{[23][34]}^2]^{2-2\varepsilon}}. \quad (39)$$

To perform the integrations efficiently, we remove the  $s_3$  variable with the  $\delta$ -function and then rescale  $s_2 \rightarrow \bar{s}_1 s_2$  which implies  $s_3 \rightarrow \bar{s}_1 \bar{s}_2$ . The denominator admits a factorized form with two factors both linear in  $s_1$ ,  $s_1 s_2 x_{7[23]}^2 + s_1 s_3 x_{7[34]}^2 + s_2 s_3 x_{[23][34]}^2 = 2\bar{s}_1 [As_1 + B]$  with

$$A = (x_{73}^+ - \bar{t} s_2 x_{23}^+)(x_{73}^- + s \bar{s}_2 x_{34}^-), \quad B = s_2 \bar{s}_2 \bar{t} s x_{23}^+ x_{34}^-. \quad (40)$$

The next integration to be performed is with respect to  $s_1$ , which produces a hypergeometric functions  ${}_2F_1$ . However, we notice that the small- $s_1$  region yields a contribution inverse in  $B$  such that the two subsequent  $s$ - and  $t$ -integration induce divergencies. This allows us to extract the leading inverse-power behavior of the regularized integral at small  $B$  and then resum the rest for  $\varepsilon = 0$ . This gives

$$\int_0^1 \frac{ds_1 s_1^{1-\varepsilon}}{[As_1 + B]^{3-2\varepsilon}} = \frac{\Gamma(1-\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(3-2\varepsilon)} \frac{1}{B^{1-\varepsilon} A^{2-\varepsilon}} - \frac{1}{2A(A+B)^2} - \frac{1}{2A^2(A+B)} + \mathcal{O}(\varepsilon). \quad (41)$$

Then integrating over the  $s$  and  $t$  variables, we get the following representation for the integral

$$\begin{aligned} \int_0^1 dt \int_0^1 ds I_1(t, s) &= \frac{1}{16} \int_0^1 dt \int_0^1 ds \int_0^1 ds_2 s_2 \bar{s}_2 \left[ \frac{1}{A(A+B)^2} - \frac{1}{A^2(A+B)} \right] \\ &- \frac{(-2\mu^2 x_{23}^+ x_{34}^-)^\varepsilon (-2\mu^2 x_{73}^+ x_{73}^-)^\varepsilon \Gamma(1-\varepsilon)\Gamma(2-\varepsilon)}{8 x_{23}^+ x_{34}^- (x_{73}^+ x_{73}^-)^2} \frac{1}{\Gamma(3-2\varepsilon)} \int_0^1 ds_2 S_\varepsilon(s_2) T_\varepsilon(s_2), \end{aligned} \quad (42)$$

where the divergent one-dimensional contributions are given by

$$S_\varepsilon(s_2) = \int_0^1 \frac{ds}{s^{1-\varepsilon}} \frac{1}{[1 + s \bar{s}_2 x_{34}^- / x_{73}^-]^{2-\varepsilon}}, \quad T_\varepsilon(s_2) = \int_0^1 \frac{dt}{t^{1-\varepsilon}} \frac{1}{[1 - \bar{t} s_2 x_{23}^+ / x_{73}^+]^{2-\varepsilon}}. \quad (43)$$

Their  $\varepsilon$ -expansion is easy to construct and reads to order  $\mathcal{O}(\varepsilon)$ , e.g., for  $T_\varepsilon(s_2)$

$$\begin{aligned} T_\varepsilon(s_2) &= \frac{1}{\varepsilon} + \frac{x_{23}^+ s_2}{x_{73}^+ - x_{23}^+ s_2} - \ln \left( 1 - s_2 \frac{x_{23}^+}{x_{73}^+} \right) \\ &+ \varepsilon \left[ \frac{x_{23}^+ s_2}{x_{73}^+ - x_{23}^+ s_2} + \frac{2x_{73}^+ - x_{23}^+ s_2}{x_{73}^+ - x_{23}^+ s_2} \ln \left( 1 - s_2 \frac{x_{23}^+}{x_{73}^+} \right) - \frac{1}{2} \ln^2 \left( 1 - s_2 \frac{x_{23}^+}{x_{73}^+} \right) - 2\text{Li}_2 \left( s_2 \frac{x_{23}^+}{x_{73}^+} \right) \right], \end{aligned} \quad (44)$$

and the one for  $S_\varepsilon(s_2)$  being analogous with the obvious substitutions of the defining variables. With poles being extracted explicitly, the remaining integrations can be performed with Mathematica. The output is given however, in a form that involves dilogarithms with arguments depending of products of plus and minus variables. Instead of relying on known identities between the dilogarithms to simplify the result and cast it as sum of functions depending either on plus or minus variables, the use of the formalism of symbols [25, 15] becomes very instrumental for fast and efficient derivations of the sought identities. Just to give an example, we encounter the following combination of dilogarithms in the output,

$$L(u, v) = \text{Li}_2 \left( 1 + \frac{v}{u\bar{v}} \right) - \text{Li}_2 \left( 1 + \frac{v\bar{u}}{u} \right) - \text{Li}_2(\bar{u}\bar{v}), \quad (45)$$

where  $u = x_{23}^+ / x_{73}^+$  and  $v = x_{43}^- / x_{73}^-$ . In order to disentangle the  $u$  and  $v$  dependence, we calculate the symbol of the right-hand side of this identity and find after simple manipulations

$$\mathcal{S}[L(u, v)] = -\frac{v}{u\bar{v}} \otimes \left( 1 + \frac{v}{u\bar{v}} \right) + \frac{v\bar{u}}{u} \otimes \left( 1 + \frac{v\bar{u}}{u} \right) + (u + v\bar{u}) \otimes (\bar{u}\bar{v}) \quad (46)$$

$$= (u + v\bar{u}) \otimes (\bar{u}\bar{v}) + (\bar{u}\bar{v}) \otimes (u + v\bar{u}) - \bar{v} \otimes u - u \otimes \bar{v} - \bar{v} \otimes \bar{v} - \bar{u} \otimes u + v \otimes \bar{v}.$$

From here, we can immediately read off the expression for the function itself (with a potentially present additive transcendental constant fixed by comparing both sides of the equation numerically),

$$L(u, v) = \ln(\bar{u}\bar{v}) \ln(u + v\bar{u}) - \ln \bar{v} \ln u - \frac{1}{2} \ln^2 \bar{v} + \text{Li}_2(u) - \text{Li}_2(\bar{v}). \quad (47)$$

The same technique is applicable to all other terms. The sum of all terms yields expressions with arguments being functions of either  $u$  or  $v$  variables separately.

Finally, we get for the integral  $I_1$ ,

$$\begin{aligned} \int_0^1 dt \int_0^1 ds I_1(t, s) = & -\frac{(-2\mu^2 x_{23}^+ x_{34}^-)^\epsilon (-2\mu^2 x_{73}^+ x_{73}^-)^\epsilon \Gamma(1-\epsilon) \Gamma(2-\epsilon)}{8 x_{23}^+ x_{34}^- (x_{73}^+ x_{73}^-)^2} \frac{\Gamma(1-\epsilon) \Gamma(2-\epsilon)}{\Gamma(3-2\epsilon)} \left[ \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{x_{72}^+}{x_{73}^+} - \frac{1}{\epsilon} \ln \frac{x_{74}^-}{x_{73}^-} \right] \\ & + \frac{1}{8 x_{23}^+ x_{34}^- (x_{73}^+ x_{73}^-)^2} + \frac{\ln(x_{72}^+/x_{73}^+)}{16 (x_{23}^+)^2 x_{34}^- x_{73}^+ (x_{73}^-)^2} - \frac{\ln(x_{74}^-/x_{73}^-)}{16 x_{23}^+ (x_{34}^-)^2 (x_{73}^+)^2 x_{73}^-} \\ & + \frac{\ln^2(x_{72}^+ x_{73}^-/x_{73}^+ x_{34}^-)}{32 x_{23}^+ x_{34}^- (x_{73}^+ x_{73}^-)^2} + \frac{\text{Li}_2(x_{23}^-/x_{73}^+)}{8 x_{23}^+ x_{34}^- (x_{73}^+ x_{73}^-)^2} + \frac{\text{Li}_2(x_{43}^-/x_{73}^-)}{8 x_{23}^+ x_{34}^- (x_{73}^+ x_{73}^-)^2}. \end{aligned} \quad (48)$$

The calculation of the second contribution  $I_2$  is much simpler since all one is after is the double and single pole part of the integral since they get compensated by the overall factor of  $\epsilon$ . Then in the analogous to Eq. (41) integral with respect to  $s_1$ , one has to keep only the first term. The subsequent integrations over  $s$  and  $t$  like done above in Eq. (44), immediately gives the final answer

$$\int_0^1 dt \int_0^1 ds I_2(t, s) = \frac{(-2\mu^2 x_{23}^+ x_{34}^-)^\epsilon (-2\mu^2 x_{73}^+ x_{73}^-)^\epsilon \Gamma^2(1-\epsilon)}{4 x_{23}^+ x_{34}^- x_{73}^+ x_{73}^-} \frac{\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)} \left[ \frac{1}{\epsilon} - \frac{x_{72}^+}{x_{23}^+} \ln \frac{x_{72}^+}{x_{73}^+} + \frac{x_{74}^-}{x_{43}^-} \ln \frac{x_{74}^-}{x_{73}^-} \right]. \quad (49)$$

Summing both contributions together, we find half of the result displayed in Eq. (24). The other half is given by the mirror symmetric diagram, computed via the formalism outlined above.

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